Triangle Wave Drive

In the mutual inductance lab report you were asked to explain the shape of Fig. 5 (reproduced below) using Faraday's Law. It sounded like a lot of people were confused about this so I have prepared this note to discuss some of the physics. It's worth thinking about this circuit in detail on your own because there are a few subtle points which can be quite confusing.

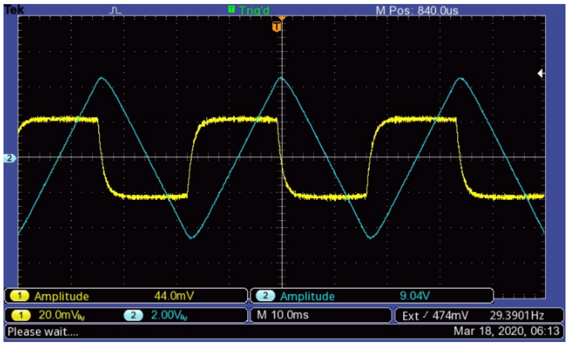

Figure 5: Induced voltage on small coil (yellow trace) when large coil is driven by a triangle-wave current (blue trace) at 30 Hz.

Before performing an experiment, it's often instructive to predict the results from theory. Let's first calculate what we should expect to see if the primary coil is driven by a triangle wave current. The system consists of a small (secondary) coil centered in a large (primary) coil as shown in Fig. 1. If there is a current going, say, counter-clockwise in the primary then there will be a magnetic field penetrating the area enclosed by the secondary yielding a magnetic flux $\Phi_s = N_s \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}_s$. Importantly, this magnetic field is *directly proportional* to the current I_p in the primary and thus so is the flux:

$$\boxed{\Phi_s \propto I_p.} \tag{1}$$

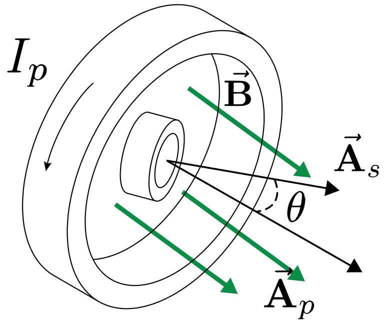

Figure 1: Relative orientation of primary coil and secondary coils. Current I_p flowing CC in the primary coil produces a magnetic field $\vec{\mathbf{B}}$ whose flux through the secondary coil is $\Phi_s = N_s \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}_s$.

By Faraday's Law the emf induced in the secondary coil by the time rate change of the magnetic flux satisfies

$$\boxed{\varepsilon_s = -\frac{d\Phi_s}{dt} \propto -\frac{dI_p}{dt}.} \tag{2}$$

The picture you should have in your head when interpreting eqn. (3) is that if the current through the primary coil is (say) counter-clockwise and it is *increasing in this direction*, as in Fig. 2, then the current in the secondary coil must be clockwise to oppose the change in magnetic flux.

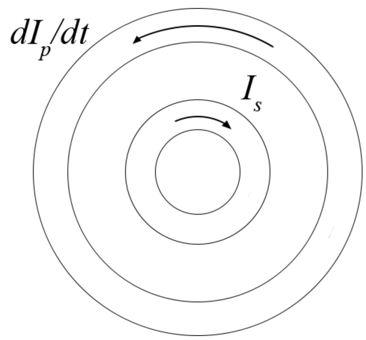

Figure 2: The emf ε_s induced in the secondary coil produces a current in the opposite direction of dI_p/dt to oppose the change in magnetic flux.

Now there are only two things we need: (i) a way to relate I_p with the voltage across the primary loop resistor V_R and (ii) a model for how the voltmeter measures the voltage across the inductor. For now I will go through these quickly, but there are some subtleties worth addressing which I will discuss at the end of this note.

(i) To relate I_p to V_R consider Fig. 3 which shows a circuit diagram of the setup. The potential difference across the resistor is simply $V_R = I_p R$, which is the voltage a voltmeter would read if it is attached to the circuit in parallel with its positive and negative terminals connected to points c and d respectively. Thus, $I_p \propto V_R$. Combining this result with eqn. (2) we conclude that

$$\boxed{\varepsilon_s \propto -\frac{dV_R}{dt}.} \tag{3}$$

(ii) The *absolute value* of the voltage across the secondary inductor will be $|\varepsilon_s|$[1]. The only thing we need to determine is the sign the a voltmeter would display. This depends on where we attach the positive and negative leads of the voltmeter. Referring back to Fig. 3, whether we attach the positive and negative leads of the voltmeter at points a and b, or at

[1]You should find this statement very sketchy: The electric field inside the inductor is 0 since it is made of the same material as the conducting wires. (Any nonzero $\vec{\mathbf{E}}$ would lead to infinite current.) Moreover there is a time-changing magnetic field localised in the inductor so $\vec{\mathbf{E}}$ is non-conservative and we can't define a potential V. From these considerations it is not clear what a voltmeter should read if we connect one across the inductor in the secondary loop. We will pick this up later.

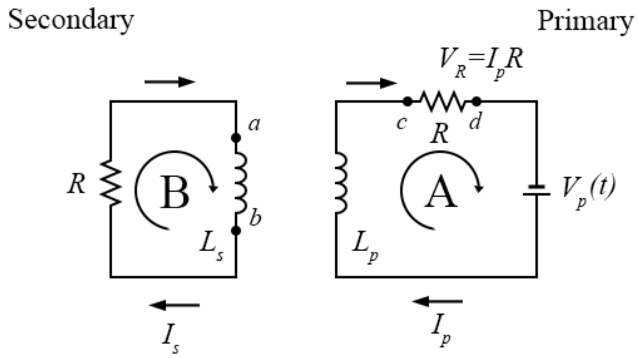

Figure 3: Diagram of coils (other circuit components hidden for clarity.)

b and a, we will observe opposite signs. This means that depending on how the voltmeter is connected it will read either $V_{L_s} = +\varepsilon_s$ or $V_{L_s} = -\varepsilon_s$. The most natural choice would be to connect the voltmeter so that it reads ε_s.

With this convention, by eqn. (3) we expect that V_{L_s} should have the opposite sign of dV_R/dt. Since V_R varies as a triangle wave, it always has a *constant slope* whose sign changes at regular intervals. So we expect that when V_R is increasing linearly, V_{L_s} should be constant with a negative sign. When dV_R/dt abruptly changes sign then we expect V_{L_s} to do the same. If we look at Fig. 5, we see that V_{L_s} and dV_R/dt share the same sign, which can be explained by having the leads of the voltmeter swapped.

Voltage across an inductor

In the footnote on the previous page I warned you that there is something *off* about saying there is a potential difference across an inductor. Potentials are only defined where there is no time-varying magnetic field, but the entire premise of an inductor is to have a time-varying magnetic field. Nevertheless, if you attach a voltmeter across an inductor the predictions made in the previous section hold. The Feynman lectures [1] in physics (Vol. II) have a nice explanation of this on page (22-2) with the caveat that it is assumed that the the voltmeter is attached to the leads of the inductor, which are assumed to be far from the time-varying $\vec{\mathbf{B}}$ field. The solution involves a careful analysis of the inductor using Faraday's law.

Jason Hafner from Rice has several videos explaining this on YouTube[2] (see "PHYS 102 — LR Circuits 2 - Faraday's Law is the correct approach for a Circuit with an Inductor!".)

Walter Lewin also discusses this at length in some supplemental lecture notes online [4, 3].

References

[1] R.P. Feynman, R.B. Leighton, and M. Sands. *The Feynman Lectures on Physics, Vol. II: The New Millennium Edition: Mainly Electromagnetism and Matter*. The Feynman Lectures on Physics. Basic Books, 2011. URL: `https://books.google.com/books?id=hlRhwGK4OfgC`.

[2] Jason Hafner. Phys 102 — lr circuits 2. `https://youtu.be/UNmEayHrCJg`.

[3] Walter Lewin. Non-conservative fields! `https://web.mit.edu/8.02/www/Spring02/lectures/lecsup3-15.pdf`.

[4] Walter Lewin. Self-inductance – kirchoff's law. `http://freepdfhosting.com/0813df09f5.pdf`.